

# Gauge bosons in a five-dimensional theory with localized gravity

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## Abstract

We consider the possibility of gauge bosons living in the recently proposed five-dimensional theory with localized gravity. We study the mass spectrum of the Kaluza-Klein (KK) excitations of the gauge fields and calculate their couplings to the boundaries of the fifth dimension. We find a different behaviour from the case of the graviton. In particular, we find that the massless mode is not localized in the extra dimension and that the KK excitations have sizeable couplings to the two boundaries. We also discuss possible phenomenological implications for the case of the standard model gauge bosons.

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## I. INTRODUCTION

It has been recently realized that extra dimensions can play a crucial role in the hierarchy problem. A first example was given in Ref. [1] where the weakness of the gravitational interaction versus the gauge interactions was obtained by allowing the graviton to propagate in some extra (and very large) dimensions.

An alternative scenario has been considered in Refs. [2,3]. They assumed an extra dimension compactified on a segment  $S^1/Z_2$  with two 4D boundaries. By adjusting the cosmological constant of the boundaries with the one in the bulk, they found a solution to the 5D metric of the form

$$ds^2 = e^{-2kR\phi} \eta_{\mu\nu} dx^\mu dx^\nu - R^2 d\phi^2, \quad (1)$$

where  $\mu = 1, \dots, 4$ , and  $R$  is the radius of the compact fifth-dimension parametrized by  $\phi$  with  $0 \leq \phi \leq \pi$ .  $k$  is a parameter related to the cosmological constant [2]. The metric (1) corresponds to a slice of  $AdS_5$  between the two boundaries located at  $\phi = 0$  and  $\phi = \pi$ . After reducing to 4D, one finds that the effective 4D Planck scale is given by  $M_P^2 = M_5^3(1 - e^{-2kR\pi})/k$  where  $M_5$  is the 5D Planck mass, the fundamental scale in the theory. Masses on the 4D boundary at  $\phi = \pi$ , however, appear to be reduced by the exponential factor  $e^{-kR\pi}$  of the induced metric on the boundary. Therefore, for  $kR \simeq 12$  and  $k \sim M_5$ , one can generate two scales for the fields living on the boundary at  $\phi = \pi$ : the Planck scale  $M_P \simeq M_5$  and the electroweak scale  $M_P e^{-kR\pi} \sim \text{TeV}$ . Fields living in the 5D metric (1) have a very peculiar Kaluza-Klein (KK) decomposition. The masses of the KK are exponentially suppressed  $M_P e^{-kR\pi} \sim \text{TeV}$  and therefore are very light even though the radius of compactification is (approximately) of Planckian size. These KK modes have extensively been studied for the case of the graviton [2,4] and a scalar field [5].

In this letter we want to consider the case of a gauge boson propagating in the  $AdS_5$  slice described above. We will study the KK decomposition of the gauge boson field and analyze

its phenomenological implications. We will show that, unlike the graviton or the scalar, the zero mode of the gauge boson is not localized in the 5D theory. Similar to the case of a flat extra dimension, this massless gauge boson spreads over the extra dimension. Also their corresponding KK excitations behave differently from those of the graviton or scalar. Although the KK masses are also exponentially suppressed, their couplings to the fields on the boundary at  $\phi = 0$  are not suppressed. They can therefore lead to sizeable effects. On the boundary at  $\phi = \pi$ , we will show that the coupling of the KK excitations depends linearly on  $\sqrt{kR}$  and therefore the KK become strongly coupled for  $kR \sim 12$ .

## II. KK DECOMPOSITION OF A 5D GAUGE BOSON

The equation of motion of a U(1) gauge boson  $A_N$  of mass  $M$  in a curved space is given by

$$\frac{1}{\sqrt{g}}\partial_M\left(\sqrt{g}g^{MN}g^{RS}F_{NS}\right)-M^2g^{RS}A_S=0, \quad (2)$$

where  $g = \text{Det}(g_{MN})$ ,  $F_{MN}$  is the gauge-field strength and we denote with capital Latin letters the 5D coordinate  $M = (\mu, \phi)$ . For the metric of eq. (1), eq. (2) leads to \*

$$\left[\eta^{\rho\nu}\partial_\rho\partial_\nu+\frac{1}{R^2}\partial_\phi e^{-2kR\phi}\partial_\phi-e^{-2kR\phi}M^2\right]A_\mu=0. \quad (3)$$

We can decompose the 5D field as

$$A_\mu(x^\mu, \phi) = \sum_n A_\mu^{(n)}(x^\mu) \frac{f_n(\phi)}{\sqrt{R}}, \quad (4)$$

where  $f_n$  satisfies

$$-\left[\frac{1}{R^2}\partial_\phi e^{-2kR\phi}\partial_\phi+e^{-2kR\phi}M^2\right]f_n=m_n^2f_n. \quad (5)$$

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\*For the massless case  $M = 0$ , we consider the gauge  $\partial_\mu A^\mu = 0$  and  $A_\phi = 0$ .

The  $m_n$  correspond to the masses of the KK excitations  $A_\mu^{(n)}$ . These masses and the corresponding eigenfunctions  $f_n$  can be found solving eq (5) with the boundary condition imposed by the orbifold  $S^1/Z_2$  (this requires  $f_n(\phi) = f_n(-\phi)$ , and that  $f_n$  and its derivative are continuous at  $\phi = 0$  and  $\phi = \pi$ ). We find

$$f_n = \frac{e^{kR\phi}}{N_n} \left[ J_\alpha\left(\frac{m_n}{k}e^{kR\phi}\right) + b_\alpha(m_n) Y_\alpha\left(\frac{m_n}{k}e^{kR\phi}\right) \right], \quad (6)$$

where  $J_\alpha$  and  $Y_\alpha$  are the Bessel function of order  $\alpha$  with  $\alpha = \sqrt{1 + M^2/k^2}$  and  $b_\alpha(m_n)$  are coefficients that fulfil

$$b_\alpha(m_n) = -\frac{J_\alpha\left(\frac{m_n}{k}\right) + \frac{m_n}{k}J'_\alpha\left(\frac{m_n}{k}\right)}{Y_\alpha\left(\frac{m_n}{k}\right) + \frac{m_n}{k}Y'_\alpha\left(\frac{m_n}{k}\right)}, \quad (7)$$

and

$$b_\alpha(m_n) = b_\alpha(m_n e^{kR\pi}). \quad (8)$$

$N_n$  is a normalization factor defined such that

$$\int_0^\pi f_n^2 d\phi = 1. \quad (9)$$

For  $kR \gg 1$ , we have

$$N_n^2 \simeq \frac{e^{2kR\pi}}{2kR} J_\alpha^2\left(\frac{m_n}{k}e^{kR\pi}\right) \simeq \frac{e^{kR\pi}}{\pi R m_n}. \quad (10)$$

From the condition (8), one can obtain the eigenmasses  $m_n$ . Let us consider separately the case of a 5D massless vector boson ( $M = 0$ ) and a massive one ( $M \neq 0$ ).

### A. Massless 5D gauge boson ( $M = 0$ case)

In this case the lowest mode is a massless state  $A_\mu^{(0)}$  with

$$f_0 = \frac{1}{\sqrt{\pi}}. \quad (11)$$

This is a constant mode. Therefore, unlike the graviton case, it is not localized in the fifth dimension. It couples with equal strength to the two boundaries  $g = g_5/\sqrt{\pi R}$  where  $g_5$  is the 5D gauge coupling. Since we are assuming that  $R$  is close to  $k \sim M_P$ , this massless mode has a coupling constant of order unity.

To obtain the massive modes, we first notice that  $\alpha = 1$  and eq. (7) reduces to

$$b_1(m_n) = -\frac{J_0(\frac{m_n}{k})}{Y_0(\frac{m_n}{k})}. \quad (12)$$

For  $m_n \ll k$ , we have

$$b_1(m_n) \simeq -\frac{\pi/2}{\ln(m_n/2k) + \gamma}, \quad (13)$$

where  $\gamma$  is the Euler constant. The KK masses can be derived from eq. (8) that now takes an approximate form:

$$J_0(\frac{m_n}{k}e^{kR\pi}) \simeq 0. \quad (14)$$

The solution are for  $n = 1, 2, \dots$

$$m_n \simeq (n\pi - 0.7)k e^{-kR\pi}. \quad (15)$$

We see then that the masses of the KK modes are exponentially suppressed and thus  $m_n \sim \text{TeV}$  (for the case  $kR \sim 12$  that we are considering). To know how they couple to the boundaries, we must look at their eigenfunctions  $f_n$  near the boundaries. For the boundary at  $\phi = 0$ , the  $f_n$  are well approximated by the second term in eq. (6):

$$f_n(\phi = 0) \simeq \frac{1}{N_n} b_1(m_n) Y_1(\frac{m_n}{k}) \simeq -\frac{1}{N_n} b_1(m_n) \frac{2k}{\pi m_n}. \quad (16)$$

From eqs. (10) and (13)-(16), we obtain that the coupling of the KK modes to the fields on the boundary at  $\phi = 0$  are

$$g^{(n)} = g\sqrt{\pi}f_n(\phi = 0) \simeq g\sqrt{\frac{\pi kR}{n - 0.2}} \left( \ln \frac{n\pi - 0.7}{2} - kR\pi + \gamma \right)^{-1}. \quad (17)$$

This coupling is of order  $g$ . For the  $n = 1$  mode we get  $g^{(1)} \simeq 0.2g$  (for  $kR \sim 12$ ). Note that this is different from the graviton case, where the coupling of the light KK excitations to the boundary at  $\phi = 0$  is exponentially suppressed [2,4]. The coupling of the KK gauge bosons to the fields on the boundary at  $\phi = \pi$  is dominated by the first term of eq. (6) that leads to  $g^{(n)} \simeq g\sqrt{2\pi kR}$ . Since it grows with the radius  $R$ , we can obtain an upper bound on  $R$  by imposing that the theory is perturbative, i.e.  $g^{(n)2}/4\pi < 1$ . We get

$$R \lesssim \frac{2}{kg^2}. \quad (18)$$

This bound is quite strong and seems to disfavor this type of scenarios with  $kR \sim 12$ .

### B. Massive 5D gauge boson ( $M \neq 0$ case)

For a massive 5D vector boson, the results are slightly different. The eigenmasses are, approximately, given by

$$J_\alpha\left(\frac{m_n}{k}e^{kR\pi}\right) + \frac{m_n}{k}e^{kR\pi}J'_\alpha\left(\frac{m_n}{k}e^{kR\pi}\right) \simeq 0, \quad (19)$$

that leads again to exponentially suppressed masses,  $m_n \sim ke^{-kR\pi} \sim \text{TeV}$ . An important difference from the  $M = 0$  case is the coupling of these modes to the  $\phi = 0$  boundary. We have now

$$g^{(n)} \simeq -g\sqrt{\pi} \frac{2\alpha}{N_n(1 - \alpha)\Gamma(1 + \alpha)} \left( \frac{m_n}{2k} \right)^\alpha. \quad (20)$$

We see that now the coupling to the boundary at  $\phi = 0$  is very small, due not only to the exponential suppression (coming from  $N_n$ ) but also to the  $m_n/k$  suppression. The coupling to the boundary fields at  $\phi = \pi$  is found to be the same as for the massless case,  $g^{(n)} \simeq g\sqrt{2\pi kR}$ .

Finally it is interesting to point out what has happened to the massless mode that appeared in the  $M = 0$  case. This mode is still present in the theory but its mass has become of order  $M$ .

### III. PHENOMENOLOGICAL SPECULATIONS

The above analysis indicates that massless gauge bosons living in a 5D bulk with the metric (1) and  $kR \simeq 12$  have very light KK excitations (of order TeV). They couple with a strength  $\sim 0.2 g/\sqrt{n}$  to the fields on the boundary at  $\phi = 0$ , but are strongly coupled ( $g^{(n)} \simeq 8g$ ) to the fields on the boundary at  $\phi = \pi$ . Massive 5D gauge bosons also have TeV KK excitations, but they couple very weakly to the boundary fields at  $\phi = 0$ .

In this section we want to consider the possibility of having the standard model (SM) fermions localized on the 4D boundary at  $\phi = 0$  with the SM gauge bosons living in the 5D bulk <sup>†</sup>. In this case the KK gauge bosons will have non-negligible couplings to the SM fermions. This looks quite surprising, since even though the cutoff of the theory for the fields on the boundary is  $M_P$ , the KK excitations can affect low-energy processes. Bounds on  $R$  can be obtained, for example, from electroweak high-precision measurements as in Ref. [6]. We get in this case a bound on the first KK mass  $m_1 > 300\text{--}500$  GeV. Deviations in four-fermion interactions can also be used in future colliders to test these scenarios similarly to the case of

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<sup>†</sup> In this scenario supersymmetry could be needed in order to preserve the Higgs mass smaller than  $M_P$ . Our motivation here will not be the hierarchy problem, but to analyze the distinctive phenomenological implications of this scenario.

a large (TeV) extra dimension. We must stress, however, that in the scenarios considered here there is an important difference from the case of a large TeV-dimension. In this latter case, the strength of the interaction of two fermions on the boundary with a gauge boson in the bulk grows, at the classical level, with the square-root of the energy (since for energies above the first KK mass, the number of KK that mediates the interaction increases with the energy). This means that the theory becomes strongly coupled above the first KK mass, and we loose perturbativity (new physics must appear). Here, however, the coupling constant of the  $n$ th KK excitation goes as  $g/\sqrt{n}$  (see eq. (17)), and therefore the strength of the interaction remains constant. At the quantum level, however, the gauge boson propagator is sensitive to the KK excitations and it is not clear if the theory will remain perturbative or it will become strongly coupled, and therefore inconsistent. Further studies along these lines are clearly needed.

If this theory is embedded at high energies  $\sim M_P$  into a GUT group (such as SU(5)), one could expect that large proton-decay operators will be induced, since the KK excitations of the GUT gauge bosons will be very light (they will have TeV masses). Nevertheless, we have seen that the coupling of the KK excitations of a massive 5D gauge boson to the fermions on the  $\phi = 0$  boundary is small — see eq. (20). Even if we sum over the full KK tower, we get that any four-fermion operator induced by a massive 5D gauge boson field is strongly suppressed  $\sim 1/k^2$  (for a 5D gauge-boson mass  $M$  of order  $k$ ). The coupling of the SM gauge bosons to the KK excitations of the GUT fields is, however, not suppressed, since massless gauge-modes can propagate in the extra dimension. Consequently, KK excitations of the GUT fields could be produced in pairs by a Drell-Yan process. It is interesting to recall that, as we said at the end of section 2, the GUT-partners of the massless SM gauge bosons are modes with masses  $M \sim k$  (these are modes that would tend to a massless and  $\phi$ -independent mode if the VEV that breaks the GUT tends to zero). These modes seems therefore that will induce, at the one-loop level, a logarithmic dependence of the gauge coupling with the high-energy scale ( $\sim M_P$ ) as in the case of a 4D theory.



Summarizing, we have studied the behaviour of a  $U(1)$  gauge boson field propagating in a slice of  $AdS_5$ . We have analyzed its KK decomposition and find that there are very light KK excitations that couple with strength  $g$  to the fermions on the boundaries. The theory, on the boundary at  $\phi = 0$ , seems to have an interesting behaviour. A priori the cutoff scale of the theory for fermions at  $\phi = 0$  is  $M_P$ . Nevertheless, it is possible from that boundary to “see” physics at  $M_P$  since the KK modes are light and couple to the fermions with non-negligible couplings. Further studies, however, are needed in order to assure the consistency of the theory at the quantum level.

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**Note added:** While writing this paper, it appeared Ref. [7] considering also gauge bosons propagating in the metric (1).

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